# Collisionless Breakdown of Magnetic Insulation in Plasmas

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The current response of an electrically biased magnetoplasma to a short photoionizing laser pulse is considered, with emphasis on the transient state. The behavior in the approach to the asymptotic state varies from monotonic to oscillatory, depending on the ratios of the collision frequency to the cyclotron frequency for the carriers. Of interest is the case equivalent to a critically damped oscillator, which allows mapping the photonic pulse to a short electric pulse, independently of the carrier lifetime. This corresponds to transient breakdown of the magnetic insulation during photoionization. Examples and applications to gaseous and semiconductor plasmas are presented.

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Magnetic insulation is a well known plasma physics concept. It controls the current flow in a plasma subjected to crossed electric and magnetic fields. Examples include the Pedersen and Hall currents driven by the interaction of the solar wind with the planetary magnetospheres [1], the Hall effect in semiconductor plasmas [2], and the operation of plasma opening switches [3]. Of equal importance is the breakdown of magnetic insulation, which is often achieved either by increasing the carrier collision frequency $\nu$ so that it exceeds the cyclotron frequency $\Omega$, by instability-induced turbulence that gives rise to an effective collision frequency with $\nu_{\text{eff}} > \Omega$, by destroying the magnetic geometry, or by some combination of these.

The purpose of this Letter is to point out a new collisionless physical process that results in the breakdown of magnetic insulation when a plasma is subjected to a short photoionizing pulse with length $\tau < \nu^{-1}$. The development of high-power, short-pulse lasers makes the topic extremely opportune. It will be shown below that in this case the ultrashort photon pulses can be mapped into ultrashort current pulses. The results apply equally to gaseous and semiconductor plasmas. Besides its intrinsic scientific interest, the phenomenon has a wide range of applications in the area of ultrashort and optoelectronic switching [4].

The basic idea is shown schematically in Fig. 1. In the absence of an electric field and neglecting collisions, carriers photoproduced with velocity $v$ at the point $O$ would execute cyclotron gyrations about centers displaced from $O$ by the gyroradii $r = v\Omega^{-1}$, where $\Omega$ is the gyrofrequency. If a constant electric field $\mathbf{E}$ is applied parallel to the $x$ axis and perpendicular to a strong magnetic field $B$, taken to point in the $z$ direction, the particles acquire a $y$ velocity $\mathbf{E} \times \mathbf{B}/B^2$ superposed on this gyration [Fig. 1(a)]. Since carriers of both signs drift in the same direction with the same mean velocity, there is no average current in this case; i.e., the material is effectively magnetically insulated. When collisions are taken into account, the direction of the drift is rotated through an angle $\theta = \cot^{-1}\mu B$ toward the positive or negative $x$ axis, depending on the sign of the charge [Fig. 1(b)]. In steady state each species carries a current [2]

$$J_x = nqv_x \sim \mu E/[1 + (\mu B)^2] \tag{1}$$

parallel to $\mathbf{E}$, where $q = \pm e$ is the carrier charge and $\mu$ is the carrier mobility. As a function of $\mu$, $J_x$ attains its maximum at $\mu B = 1$. Magnetic insulation reduces the value of the current in comparison with that of an unmagnetized plasma by a factor $1 + \Omega^2/\nu^2 = 1 + (\mu B)^2 \gg 1$. If the carrier mobilities are unequal, there is also a net current $J_y$ in the $\mathbf{E} \times \mathbf{B}$ direction.

To see what happens in the transient stage following the creation of carriers by optical activation, consider a simple carrier equation of motion in the plane transverse to $B$:

$$m d\mathbf{v}/dt = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - m\nu\mathbf{v}\,. \tag{2}$$

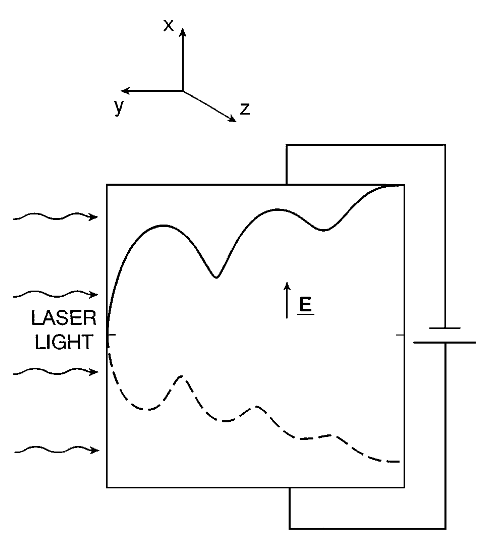


FIG. 1. Orbits of positive (solid trace) and negative (broken trace) carriers in silicon subjected to crossed field $\mathbf{E}$ and $\mathbf{B}$ for effective mass ratio $\epsilon = \sqrt{m_-/m_+} = 0.8$.

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Assuming that the carriers are born with negligible initial velocity, the solution written in terms of the complex velocity $w = v_x + iv_y$ is

$$w(t) = \frac{qE}{m} \frac{1 - \exp[-(\nu \pm i\Omega)t]}{\nu \pm i\Omega} . \tag{3}$$

Initially (for times short compared with $\nu^{-1}$ and $\Omega^{-1}$) the velocity is in the $\pm x$ direction (depending on the sign of $q$) and grows proportionally to $t$. For times greater than or of order $\Omega^{-1}$ the linear approximation of the second term ceases to be valid, and Eq. (3) describes gyration about the field lines. If the carriers originate over a period of time comparable with $\Omega^{-1}$, the individual particles become desynchronized on this time scale and the current decays due to phase mixing. After a few collision times the second term becomes negligible, and to a good approximation the late-time carrier motion is rectilinear and uniform.

Each charge passes through the same stages, but the ones created later are at an earlier stage in their evolution. The current associated with each species is therefore an integral over successive increases $dn = \dot{n}dt$ in the carrier population, given by

$$J_x(t) = q \int_0^t dt' \, \dot{n}(t') v_x(t - t') , \tag{4}$$

where $n$ is the carrier number density and the dots stands for a time derivative. Evidently the current can continue to grow as long as $\dot{n}$ is nonzero. For times short compared to the carrier loss time the number density resulting from photoproduction is given by

$$\dot{n} = n_0 Q(t) , \tag{5}$$

where $n_0$ is the final value and $Q(t)$ is the volume carrier production rate, proportional to the intensity of the laser radiation.

The sudden introduction of carriers can break the magnetic insulation. This is a new physical effect, which to our knowledge has not been discussed previously. We will show that it can take place whenever the optical pulse has a rise time short compared with $(\Omega_+\Omega_-)^{-1/2}$ where $\Omega_\pm$ is the cyclotron frequency of the carriers, even if the carrier loss time is infinite. Another way to view this process is to say that the insulation is broken by the polarization current associated with the dipole moments of the carrier pairs.

As noted above, the presence of an effective collision mechanism causes carriers of different species to drift in the $\mathbf{E} \times \mathbf{B}$ direction at different rates, as a result of which a net charge flow develops (the Hall current perpendicular to $\mathbf{E}$ and the Pedersen current parallel to it). But any deviation from charge neutrality in a plasma gives rise to strong electric fields that act to restore charge balance. The electric field therefore takes the form [5]

$$\mathbf{E}(t) = \mathbf{e}_x E_0 + \mathbf{e}_y E_y(t) , \tag{6}$$

where $E_0$ is constant and $E_y(t)$ is the polarization field required to maintain charge neutrality (the ambipolar field). The velocity automatically adjusts to make the net transverse current vanish, i.e., the ambipolar condition $n_+ v_y^+ = n_- v_y^-$ is satisfied. The ambipolar condition, which is responsible for the Hall effect in plasmas and solid conductors, determines $E_y$. If the extrinsic carrier density is much less than that due to photoproduction, then we have $n_+ = n_-$ and the velocities satisfy $v_y^+ = v_y^- \equiv v_y$.

Defining the Laplace transform by $\tilde{f}(s) = \int_0^\infty dt \, e^{-st} f(t)$, we rewrite Eq. (4) for the polarization current

$$\tilde{\mathbf{J}}(s) = s\tilde{n}(s) \sum_\pm (\pm e) \tilde{\mathbf{v}}_\pm(s) , \tag{7}$$

where $\pm$, the sign of the carrier charge, is used as a species label. Each carrier species satisfies an equation of motion of the form (2), which after Laplace transforming becomes

$$(s + \nu_\pm)\tilde{v}_x^\pm = (\pm e/m_\pm^x)(s^{-1}E_0 + \tilde{v}_y^\pm B) , \tag{8}$$

$$(s + \nu_\pm)\tilde{v}_y^\pm = (\pm e/m_\pm^y)(\tilde{E}_y - \tilde{v}_x^\pm B) . \tag{9}$$

We assume that the medium is homogeneous and neglect collisional heating. In the case of a solid-state plasma we also assume that the coordinate system is aligned with the principal axes of the effective mass tensor. The ambipolarity condition is equivalent to $\tilde{v}_y^+ = \tilde{v}_y^- \equiv \tilde{v}_y$. Solving for $\tilde{v}_y$ by eliminating $\tilde{E}_y$ from the two components of Eq. (9) and substituting in Eq. (8), we find $\tilde{v}_x^\pm$. The result simplifies considerably under the condition $m_+^x/m_+^y = m_-^x/m_-^y$, in which case Eq. (7) yields

$$\tilde{J}_x = \frac{\tilde{n}eE_0}{B} \frac{\Omega'_+(s + \nu_-) + \Omega'_-(s + \nu_+)}{(s + \nu_+)(s + \nu_-) + \Omega'_+\Omega'_-} , \tag{10}$$

where $\Omega'_\pm = eB/(m_\pm^x m_\pm^y)^{1/2}$. In what follows we will take the effective mass tensor to be isotropic ($m_\pm^x = m_\pm^y$) and omit the prime from $\Omega_\pm$.

Inverting the Laplace transform, we obtain a general expression for the time-dependent current:

$$J_x(t) = \frac{eE_0}{B} \int_0^t dt' \, n(t - t') e^{-\overline{\nu}t'} \Bigg[ (\Omega_+ + \Omega_-) \cos\overline{\Omega}t' - \frac{1}{2\overline{\Omega}} (\nu_+ - \nu_-)(\Omega_+ - \Omega_-) \sin\overline{\Omega}t' \Bigg] , \tag{11}$$

where $\overline{\nu} = \frac{1}{2}(\nu_+ + \nu_-)$ and $\overline{\Omega}^2 = \Omega_+\Omega_- - \frac{1}{4} \times (\nu_+ - \nu_-)^2$. For a delta-function source creating carriers of each species with density $n_0$ at $t = 0$, this becomes

$$\frac{J_x(t)}{J_x(\infty)} = 1 - e^{-\overline{\nu}t} \cos\overline{\Omega}t + \frac{(\nu_+^2 - \nu_-^2)(\Omega_+ - \Omega_-) + 2\overline{\Omega}^2(\Omega_+ + \Omega_-)}{4\overline{\Omega}(\nu_+\Omega_- + \nu_-\Omega_+)} \times e^{-\overline{\nu}t} \sin\overline{\Omega}t , \tag{12}$$

where

$$J_x(\infty) = \frac{n_0 eE_0}{B} \frac{\nu_+\Omega_- + \nu_-\Omega_+}{\nu_+\nu_- + \Omega_+\Omega_-} \tag{13}$$

is the asymptotic value to which the current decays. Note that $J_\infty$ is always nonzero.

We introduce the dimensionless time as $T = t_0\Omega_0$, where $\Omega_0 = \sqrt{\Omega_+\Omega_-}$, and define

$$\epsilon_+ = \sqrt{\frac{m_+}{m_-}}, \quad \epsilon_- = \sqrt{\frac{m_-}{m_+}}, \quad \alpha_\pm = \frac{\nu_\pm}{\Omega_\pm}, \tag{14a}$$

$$\nu = \tfrac{1}{2}(\alpha_+\epsilon_- + \alpha_-\epsilon_+),$$
$$\Omega^2 = 1 - \tfrac{1}{4}(\epsilon_-\alpha_+ + \epsilon_+\alpha_-)^2. \tag{14b}$$

Equation (12) can be written in the symmetric form

$$\frac{J(t)}{J(\infty)} = 1 - e^{-\nu T}\cos T + \frac{e^{-\nu T}\sin T}{2(\alpha_+ + \alpha_-)} \times \left\{\frac{\nu}{\Omega}[\alpha_+(\epsilon_-^2 - 1) + \alpha_-(\epsilon_+^2 - 1)] + \Omega(\epsilon_+ + \epsilon_-)\right\}. \tag{15}$$

Notice that $\alpha_\pm = (\mu_\pm B)^{-1}$, where $\mu$ is the carrier mobility.

Equation (12) and its equivalent equation (15) are the basic new result of this Letter. It is easy to see that the temporal character of $J_x(t)$ depends on the parameter $\nu$. For $\nu \gg 1$, $J_x(t)$ monotonically approaches $J_x(\infty)$ on a collisional time. The behavior is similar to the unmagnetized case. For $\nu \ll 1$, $J_x(t)$ performs a series of damped oscillations with frequency $\sqrt{\Omega_+\Omega_-}$ as it approaches $J_x(\infty)$. Finally for $\nu \approx O(1)$, $J_x(t)$ displays a behavior similar to a critically damped oscillator and approaches $J_x(\infty)$ after overshooting it. The amplitude of the overshoot depends on the carrier mass ratio $\epsilon_+$ and the magnetization parameters $\alpha_+$ and $\alpha_-$. This last case has important implications since it allows mapping of a short photonic pulse to an electric pulse with duration $(\Omega_+\Omega_-)^{-1/2}$, independently of the carrier loss time.

Equation (12) or (15) simplifies significantly in some limiting cases, allowing analytic exploration of the parametric dependences. First for $\epsilon_- \ll 1$, corresponding to a gaseous plasma, using the fact that $\Omega_+/\Omega_- = \epsilon_-^2 \ll 1$, while $\nu_+/\nu_- \approx \epsilon_-$ and assuming $\overline{\Omega} \gtrsim \overline{\nu}$, we find

$$\frac{J_x(t)}{J_x(\infty)} = 1 - e^{-\nu_- t/2}\cos\Omega_0 t + \frac{1}{2}\frac{\Omega_0}{\nu_+}e^{-\nu_- t/2}\sin\Omega_0 t\,. \tag{16}$$

Critically damped behavior requires $\Omega_0/\nu_- \approx O(1)$, while the overshoot depends on the ratio $\Omega_0/\nu_+$. Since $\nu_+/\nu_- \approx O(\epsilon_-)$, large overshoots are expected. This is seen from the numerical solution of the complete equation (15) for a weakly ionized hydrogen plasma for $\alpha_- = 0.04$ and $\alpha_+ = 0.4$, 1.0, 2.0 (these parameters correspond to $\nu_-/\Omega_0 = 2$ and $\nu_+/\Omega_0 = 0.01, 0.025, 0.05$). We can see that the overshoot amplitude is controlled by the ion collisionality, while the temporal behavior is controlled by electron collisionality. The behavior shown here is characteristic of strongly magnetized electrons ($\alpha_- \ll 1$), while only the amplitude depends on the value of $\alpha_+$. The actual response time varies with the magnetic field and can be found from $\Omega_0 = 4 \times 10^9 B_T\ \mathrm{sec}^{-1}$, where $B_T$ is the value of the magnetic field is expressed in tesla.

Equation (12) has a simple form also for $\Omega_+ = \Omega_-$ ($\epsilon_+ = \epsilon_- = 1$) and for $\nu_+ = \nu_-$. In either case the result is independent of the mass ratio and is given by

$$\frac{J_x(t)}{J_x(\infty)} = 1 - e^{-\overline{\nu}t}\cos\Omega_0 t + \frac{\Omega_0}{2\overline{\nu}}e^{-\overline{\nu}t}\sin\Omega_0 t\,. \tag{17}$$

This is characteristic of semiconductor plasma behavior. Notice that if we require behavior resembling critical damping, the overshoot amplitude is limited to a factor of 2. Higher overshoot amplitudes can be achieved only at the expense of having potentially undesirable damped oscillations. Such behavior is seen in Fig. 3, which shows the case of a photoionized PbTe semiconductor at plasma room temperature. For PbTe the effective masses are $m_+^* \approx m_-^* \approx 0.18 m_e$ and the mobilities $\mu_- = 0.6\ \mathrm{m^2/V\,sec}$, $\mu_+ = 0.4\ \mathrm{m^2/V\,sec}$. The results are shown for four values of magnetic field $B = 1$, 3, 5, 10 T, which correspond to $\alpha_- = 1/\mu_- B = 1.67$, 0.56, 0.33, 0.17 and $\alpha_+ = 1/\mu_+ B = 2.5$, 0.83, 0.5, 0.25. These values of the magnetic field correspond to $\Omega_0 \approx 1.1 \times 10^{12}$, $3.3 \times 10^{12}$, $5.5 \times 10^{12}$, $11 \times 10^{12}\ \mathrm{sec}^{-1}$, and the response times are in the subpicosecond range. Figure 4 shows the results for photonionized Ge at 70 K, for which $\nu_+ = \nu_-$ holds. In this case we have $\mu_+ = 6\ \mathrm{m^2/V\,sec}$, $\mu_- = 3\ \mathrm{m^2/V\,sec}$. Results are shown for $B = 0.25$, 0.5, 0.75, and 1.0 T. In the last case the value of $\Omega_0 \approx 3 \times 10^{12} B_T\ \mathrm{sec}^{-1}$.

The results shown in Figs. 2–4 are characteristic of the range of the responses expected in various plasma media. It is clear that gaseous plasmas with $\epsilon_- \ll 1$ can produce large overshoots with response times in the subnanosecond range for magnetic fields in the 1 T range. On the other hand, semiconductor plasmas produce moderate overshoots. However, their response time is in the subpicosecond range (and even femtosecond) range for materials with small effective mass. Equation (15)

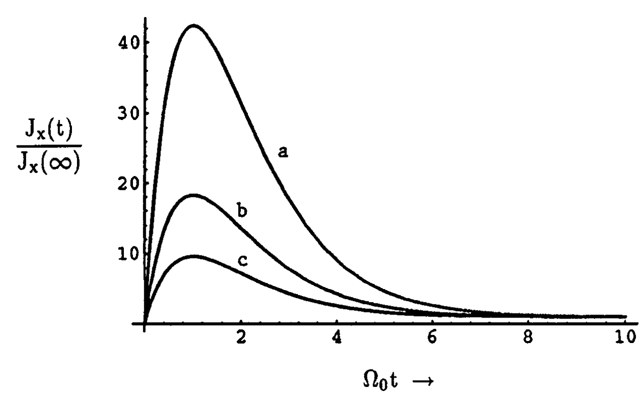


FIG. 2. Temporal behavior of a hydrogen plasma to a $\delta$-function photoionizing pulse; $\nu_-/\Omega_- = 0.04$ and $\nu_+/\Omega_+ = 0.4$ (a), 1 (b), 2 (c).

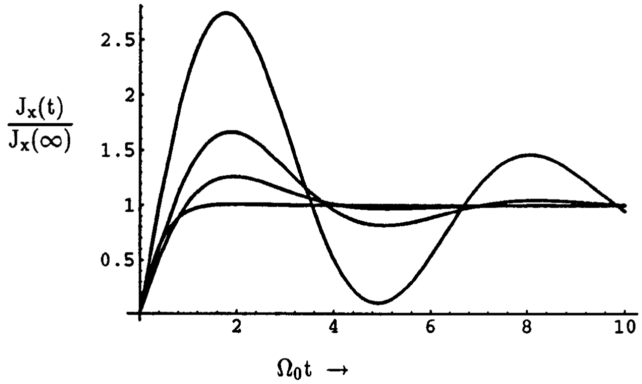


FIG. 3. Response of a PbTe photoconductor to a $\delta$-function ionizing pulse for values of $B = 1, 3, 5, 10$ T. The unit on the horizontal axis varies with the magnetic field as $\Delta t = 0.9 B_T^{-1}$ psec. Representative case of $\epsilon_+ \approx \epsilon_-$.

can be used to determine the semiconductor material and required temperature at which a particular photoelectronic response is produced.

In closing we note that it is important to differentiate between the physics described in this Letter and a related preliminary comment published recently [6]. The comment presented a theoretical explanation of an experiment in which the response time of an optically controlled photoconducting Si switch activated by a long laser pulse with $\tau \gg 1/\nu_-$ was significantly reduced by the presence of a magnetic field when $\Omega_- < \nu_-$. The theoretical model neglected the ambipolar field $E_y$ in Eq. (6). As a result, the positive and negative carriers were uncoupled and a pure electron model was used to compute the current response. For the long-time collisional regime emphasized in the comment this is a good assumption, since no net current is induced by the presence of the $E_y$ field. However, care should be exercised in neglecting the ambipolar fields and using a simplistic pure electron model in the short time scale ($\tau < 1/\nu_-$) collisionless regime. While such a model indicates a small collisionless overshoot due to electron ballistic effects alone, it predicts the wrong response time scale $\Omega_-$ rather than $\Omega_0$, a maximum overshoot of less than 2 for the critically damped case as compared to the value of over 40 seen in Fig. 2, and does not produce the correct scaling laws relevant to proof-of-principle experiments and fabrication of appropriate materials. It can be seen from the present analysis, which includes both ballistic and ambipolarity effects, that a pure electron model is correct only in the singular and unrealistic case of $\Omega_- = \Omega_+$ and $\nu_- = \nu_+$.

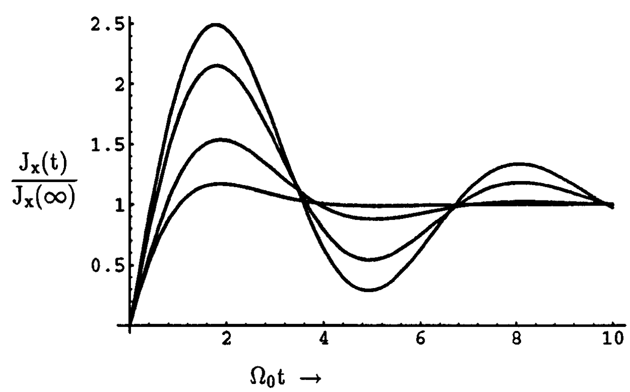


FIG. 4. Same as Fig. 3 for Ge at 70 K, for $B = 0.25, 0.5, 0.75, 1$ T. In this case $\Omega_0 = 3 \times 10^{12} B_T$.

In summary, we have presented a theoretical analysis of the response of electrically biased magnetoplamsas to a short ionizing pulse with sufficient energy to modify the conductivity significantly in a time short compared to $1/\Omega_0$. The analysis emphasized the effects of transients. It was shown for the first time that the magnetic insulation can be broken during the ionization time for a particular combination of mobilities and magnetic fields. This physical process can generate extremely short current pulses even when the carrier loss times are long. Applications of the technique are in the areas of fast switching for pulsed power and computer applications, photodetection, and development of powerful microwave sources in the teraherz range [4,7]. These topics are currently under study and will be presented in the appropriate applied forums.

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